

o So we will deny this!

For z_i, z_{i+1} : if $l \in [h(z_i), h(z_{i+1}))$ split q_{z_i} & split $q_{z_{i+1}}$
 \hookrightarrow let \tilde{q}_{z_i} take 1 of l ; else keep q_{z_i}
 \hookrightarrow let $\tilde{q}_{z_{i+1}}$ take 0 of l ; else keep $q_{z_{i+1}}$ } else they already disagree

o Then split $\tilde{q}_i = B \setminus h(i)$ is still in u ; i.e. $\tilde{q} \in P(\Sigma \setminus \{0, u\})$.

o But now $\tilde{q} \Vdash \left[\bigcap_{i \in \omega} (A_i \cup f(i)) \cap (B \setminus h(0)) = \emptyset \right]$

\hookrightarrow Take $m \in B \setminus h(0)$, i.e. $m > h(0)$.

\hookrightarrow find n s.t. $h(z_i) \leq m \leq h(z_{i+1})$

$\hookrightarrow \tilde{q}_{z_i}$ or $\tilde{q}_{z_{i+1}}$ force $m \notin A_i \cup f(i)$ $\left[\begin{array}{l} \text{split}(q_i) \subseteq B \setminus h(i) \\ \& f(i) \subseteq h(i) \text{ by } f \leq h \end{array} \right]$

\hookrightarrow i.e. $m \notin \bigcap_{i \in \omega} (A_i \cup f(i))$

\hookrightarrow i.e. $\tilde{q} \Vdash \bigcap_{i \in \omega} (A_i \cup f(i)) \cap B \setminus h(0) = \emptyset$

o This contradiction concludes the proof — of the simplified case.

o What about the " $x_i \in \tilde{u} \forall i$ " assumption?

\hookrightarrow If $\forall i x_i \in \tilde{u}$ or $\forall i x_i \notin \tilde{u}$, use the same argument mirror

\hookrightarrow otherwise $\exists i x_i \in \tilde{u}$

\hookrightarrow Define intervals I_n such that $\exists j_1 \neq j_2 \in I_n = [x_{j_1}, x_{j_2}] \in \tilde{u}$

\hookrightarrow Without loss, $(I_n)_{n \in \omega} \in V$ [priorize using ω -bounding] $\left[\begin{array}{l} \text{used to be } \\ I_n = [z_n, z_{n+1}] \end{array} \right]$

\hookrightarrow Define $A_n = \bigcap_{i \in I_n} \{x_i\}$ (which are in \tilde{u})

\hookrightarrow Proceed as before $\left[\begin{array}{l} \text{used to be } x_{z_n} \wedge x_{z_{n+1}} \\ \text{these } j_1, j_2 \end{array} \right]$

\hookrightarrow Manipulate q_i for $i \in I_n$ (not just z_n, z_{n+1}) to disagree on $x_{j_1} \wedge x_{j_2}$

\hookrightarrow Get the same contradiction.

□