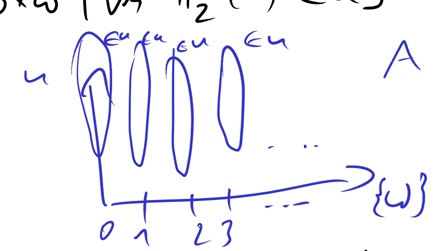
A small detour $G(u)^\omega$ & $G(\omega\otimes u)$ & $S(\omega\otimes u)$

It turns out that we have to work a little harder. Remember that we want to destroy P-points. Of course, $G(u)$ and $S(u)$ destroy u. The question is, if we iterate this process for all P-points, we might "resurrect" u later in the sense that "$\exists\, \tilde{u} \supseteq u$, $\tilde{u}$ P-point" could hold at a later stage of the iteration.

As far as I know it's not known whether $G(u)$ and $S(u)$ can prevent this.
But Shelah showed that something closely related does work!

(1) Given a filter u, we are interested in the filter $\{\omega\}\otimes u = \{A \subseteq \omega\times\omega \mid \forall n\ \pi_2^{-1}(n) \in u\}$

the usual tensor product of filters



(2) Note that if u is a P-filter, $\{\omega\}\otimes u$ has a base of the form $\left(\prod_{i\in\omega}\{i\}\times(A\setminus f(i))\right)_{A\in u,\ f\in\omega^\omega}$

[simply take a pseudo intersection of the columns]

(3) In fact, if u is a non-meagre P-filter, so is $\{\omega\}\otimes u$.

[We skip the (straight forward) proof; see Shelah Proper & Improper Forcing]

(4) In particular, if u is a non-meagre P-filter, $G(\{\omega\}\otimes u)$ & $S(\{\omega\}\otimes u)$ are proper & ω^ω-bounding!

The advantage of using $\{\omega\}\otimes u$ lies in the natural (and well-behaved) sequence of reals that arise from the fibres of the generic real (which lives on $\omega\times\omega$).
I should mention that for Grigorieff forcing, Shelah's book was yet another simplification — it turns out that $G(\{\omega\}\otimes u) \cong G(u)^\omega$ (which is very easy).
We won't need this and in fact, $\{\omega\}\otimes u$ has more advantages in the long run.