What do we get?

- After a match in the proper game,

  I has produced $(\dot{\alpha}_n)_{n\in\omega}$ with $\Vdash \dot{\alpha}_n \in ON$

  II has produced $(B_n)_{n\in\omega}$ $B_n \subseteq ON$

  as well as a strategy for I in the P-point game

- Now u is a non-meagre P-filter, i.e. the P-filter game strategy can be beaten.
- In other words, there are $(a_i)_{i\in\omega}$, $(p_{(a_0,\dots,a_i)})_{i\in\omega}$, $(H_{\langle a_0,\dots,a_i\rangle})_{i\in\omega}$ as described above

  and $\bigcup_{i\in\omega} a_i \in u$

- But then (as in the last lemma) fusion is possible, i.e.

$$q := \bigcap_{i\in\omega} p_{(a_0,\dots,a_i)} \in \mathbb{P} \quad ; \text{ of course } q \leq p$$

and $q \Vdash \dot{\alpha}_i \in H_{(a_0,\dots,a_i)}$

- In particular, $q \Vdash \dot{\alpha}_n \in B_n$ — so II wins the proper game. $\square$

After these important, but technical lemmas, we can approach the key lemma, the combinatorial argument that after killing a P-point with Grigorieff forcing, we will not resurrect the P-point in a later ω^ω-bounding extension.

Unfortunately, Grigorieff forcing is not enough for this. [AFAIK]

Shelah's original argument uses $G(u)^\omega$, the full ω-product of the Grigorieff forcing with a non-meagre P-filter.