Definition $\mathbb{P}$ is proper iff II has a winning strategy in the proper game for every $p \in \mathbb{P}$

The proper game (below $p \in \mathbb{P}$)

I	$\dot\alpha_0$ with $\Vdash \dot\alpha_0 \in ON$	...	
II	countable $B_0 \subseteq ON \cap V$	...	

II wins iff $\exists q \le p : q \Vdash \forall n \ \dot\alpha_n \in \bigcup_{k \in \omega} B_k$

Lemma If $\mathbb{P}$ is $G(u)$ or $S(u)$ and u is a non-meagre P-filter, then $\mathbb{P}$ is proper.

Proof: This is a little twisted: Our strategy for II in the proper game is to build a strategy for I in the P-point game. The strategy for the P-point game is build exactly the way we built it in the previous lemma.

- Fix $p \in \mathbb{P}$
- After playing up to n

I	...	$\Vdash \dot\alpha_n \in ON$	$\Vdash \dot\alpha_{n+1} \in ON$
II	...	B_n	?

we have built on the side a partial strategy for I in the P-point game

I	split(p)		...	split$(p_{(a_0,\dots,a_n)})$?
II		$a_0 \subseteq$ split(p)	...	a_{n+1}

as well as $H_{(a_0,\dots,a_i)} \subseteq_f ON \cap V$

such that $p_{(a_0,\dots,a_i)} \Vdash \dot\alpha_i \in H_{(a_0,\dots,a_i)}$

$p_{(a_0,\dots,a_i)} \le p_{(a_0,\dots,a_j)}$ $\forall i \ge j$

For all possible moves by II in this P-point game so far, derived from all moves $\dot\alpha_i$ in the proper game.

- As before, thanks to the "crucial lemma" this is easy to continue for every $(a_0,\dots,a_{n-1})^\frown(a_n)$
- So we get $p_{(a_0,\dots,a_{n+1})}$, $H_{(a_0,\dots,a_{n+1})}$ (for all continuations by II)
- Back in the proper game, II now plays all $H_{(a_0,\dots,a_{n+1})}$:

$$B_{n+1} := \bigcup \{ H_{(a_0,\dots,a_{n+1})} \mid (a_0,\dots,a_{n+1}) \text{ a play of II against our partial strategy} \}$$

- Of course, B_{n+1} is countable.