Imagine we played as follows

I \ \text{split}(p_0) \ \text{split}(p_1) \ \ldots
II \ a_0 \text{split}(p_0) \ a_1 \text{split}(p_1) \ \ldots
\text{with} \ p_n \downarrow j_{n+1} = j_n

Then if \( a \) is a non-negative split, I could beat the strategy!
In that case \( U \backslash a \equiv e_n \) - if only we could prevent the splitting levels from surviving, i.e., \( U \equiv \text{split}^n \), then \( \gamma_n \ racer \) could be a condition!

Of course we cannot hope for this - just imagine if \( \gamma \) was the greatest real!

But we need far less anyway...

\[ \text{Lemma} \quad p \in P, p \vdash \exists x \in V, s \equiv \text{split}^p \Rightarrow \exists x \in V, q \in P : q \vdash x \in H \]

\[ \exists s \equiv \text{split}^q \]

\[ \text{Proof: Let's prove this by picture (this is simply a diagram...)} \]

Take each subtree \( q \in P \) [there are finitely many] say \( j \)-many.

Refine each \( p_i \) to \( q_i \leq p_i \) to have \( q_i \vdash x = x_i \) for some \( x_i \in V \).

Then \( q = \bigcup_{k_j} q_j \leq p \) and \( q \vdash x \in \{ x : p \vdash x \} \).