Theorem \( \mathfrak{u} \) is a non-meagre \( \mathbf{P} \)-filter if and only if \( \mathfrak{u} \) has no winning strategy in the \( \mathbf{P} \)-point game.

Proof: Not relevant of further explanation.

The \( \mathbf{P} \)-point game: For all filters \( \mathfrak{u} \) on \( \mathbb{N} \),

\[
\begin{align*}
& \text{I} & A_0 \in \mathfrak{u} & A_1 \in \mathfrak{u} & \ldots & \text{II wins if} \\
& \text{II} & \xi \in A_0 & \xi \leq A_1 & \ldots & \nu \in \mathfrak{u} \ni \\
\end{align*}
\]

Note: II cannot have a winning strategy.

[Try playing two games, alternating to get \( \mathfrak{u}_1 \cap \mathfrak{u}_2 = \emptyset \).]

I. Basic properties

If \( \mathfrak{u} \) is a non-meagre \( \mathbf{P} \)-filter, both forces are proper \( \mathfrak{w} \)-bounding.

Towards these goals, let's consider our strategy. Since it's conceptually easier, let's start with \( \mathfrak{w} \)-bounding.

Given \( \mathfrak{u} \) with \( \mathfrak{u} \subseteq \mathbb{N}^\mathbb{N} \), we can, of course, use the usual forcing lemma to get conditions

\[
\left( \mathfrak{a} \ni \xi \right) \text{ with } \mathfrak{a} \subseteq \mathbb{N}^\mathbb{N} \text{ and } \mathfrak{a} \cap \mathfrak{u} = \emptyset.
\]

But our forcing isn't \( \mathfrak{w} \)-closed, so we need some kind of fusion.

This is where the game comes in.