<u>Theorem</u> u non-meagre P-filter iff I has no winning strategy in the P-point game

Proof: Not relevant; cf. Shelah, Laflamme

<u>The P-point game</u>: For a filter u on ω

I	$A_0 \in u$	$A_1 \in u$	...	II wins iff
II	$a_0 \subseteq_f A_0$	$a_1 \subseteq A_1$	...	$\bigcup_{i\in\omega} a_i \in u$

<u>Note</u> II cannot have a winning strategy
[I can play two games, alternating, to get $\bigcup_{i\in\omega} a_i \cap \bigcup_{i\in\omega} \tilde{a}_i = \emptyset$]

I. Basic properties

If u is a non-meagre P-filter both forcings are proper & ω^ω-bounding.

Towards these goals, let's consider our strategy. Since it's conceptually easier let's start with ω^ω-bounding.

Given $\dot{f}$ with $\Vdash_{\mathbb{P}} \dot{f} \in \omega^\omega$, we can, of course, use the usual forcing lemma to get

conditions $\left.\begin{matrix} p_{n+1} \leq p_n \\ k_n \in \omega \end{matrix}\right\}$ with $p_n \Vdash \dot{f}(n) = k_n$.

But our forcing isn't σ-closed; so we need some kind of fusion.

This is where the game comes in.