Grigorieff Forcing, Sacks Forcing & Shelah's model without P-points

<u>Definition</u> For u filter on ω

$$G(u) = \{ f : X \to 2 \mid \omega \setminus X \in u \}; \quad f \le_{G(u)} g \iff f \supseteq g$$

$$S(u) = \{ T \subseteq {}^{<\omega}2 \mid \operatorname{split} T \in u \}$$

with $\operatorname{split} T :=$ complete splitting levels
$= \{ n \in \omega \mid \forall s \in T : |s| = n \Rightarrow s^\frown 0, s^\frown 1 \in T \}$

$$T' \le_{S(u)} T \iff T' \subseteq T$$

<u>Note</u>: $G(u) \hookrightarrow S(u)$

$f \longmapsto T_f :=$ "splitting levels $\omega \setminus \operatorname{dom} f$, else like f"
$= \{ s \in {}^{<\omega}2 \mid \forall n \in \operatorname{dom} s \cap \operatorname{dom} f : s(n) = f(n) \}$

So we'll think of $G(u) \subseteq S(u)$, i.e., we prefer trees

We're interested in P-points, more generally P-filters.

<u>Def</u>: u is a P-filter [for $u \in \omega^*$ say P-point] iff $\forall (A_n)_{n \in \omega}$ in u $\exists A \in u \; \forall n \; A \subseteq^* A_n$
"pseudo intersection" i.e. $|A \setminus A_n| < \omega$

[for technical reasons]

u is non-meagre iff $\forall f : \omega \to \omega$ fin-1 : $f(u) = \langle f[A] \mid A \in u \rangle \neq$ Fréchet filter